A Thrice Randomised Ergodic Algorithm for a Multiple Access System with a Partial Binary Feedback

Mikhail Chebunin

Sobolev Institute of Mathematics of SB RAS and Novosibirsk State University, Novosibirsk, Russia

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Abstract

We consider a decentralised multiple access model with an infinite number of users, a single transmission channel, and an adaptive transmission protocol that does not use the individual history of messages. There is no exchange of information between the users, and with any such protocol, each message present in the system in time slot $[n, n+1)$ is sent to the channel with probability p_n that depends on the history of feedback from the transmission channel, independently of everything else.

Background

It is known since the 80's (see e.g. [1], [2]) that with ternary feedback "Empty-Success-Collision" the channel capacity is e^{-1}: if the input rate is below e^{-1}, then there is a stable transmission protocol; and if the input rate is above e^{-1}, then any transmission protocol is unstable. By the ternary feedback, we mean the following: the users can observe the channel output and distinguish among three possible situations: either no transmission ("Empty") or transmission from a single server ("Success") or a collision of messages from two or more users ("Conflict").

A stable protocol may be constructed recursively as follows: given probability p_n in time slot $(n, n+1)$ and a feedback at time $n+1$, probability p_{n+1} is bigger than p_n if the slot $(n, n+1)$ was empty, $p_{n+1} = p_n$ if there was a successful transmission, and p_{n+1} is smaller than p_n if there is a conflict.

Similar results (existence/nonexistence of a stable protocol if the input rate is below/above e^{-1}) hold (see e.g. [3], [4]) for systems with either of two binary feedbacks, "Empty-Nonempty" (a user cannot differentiate "Success" and "Failure") or "Failure-Nonfailure" (a user cannot differentiate "Success" and ""Empty").

The third type of protocols, with "Success-Failure" binary feedback has a different nature. Until recently (see [5]), there were known only stable protocols that use an extra information (testing package, individual history of messages, etc.)

Foss, Hayek and Turlikov ([6]) introduced a new "doubly stochastic" protocol for stability of a system with the "Success-Failure" feedback. They showed that for any pair of numbers $0 < \lambda_0 < \lambda_1 < e^{-1}$ exists a class of protocols that make stable a system with input rate λ, for any $\lambda \in [\lambda_0, \lambda_1]$. However, any such protocol depends explicitly on λ_0 and λ_1. Chebunin ([7]) proved existence of stable "doubly randomised" protocols that do not depend on value $\lambda \in (0, e^{-1})$, and estimated the rate of convergence to stationarity. A general form of such protocols was suggested in [6], but without specification of auxiliary functions.

Description

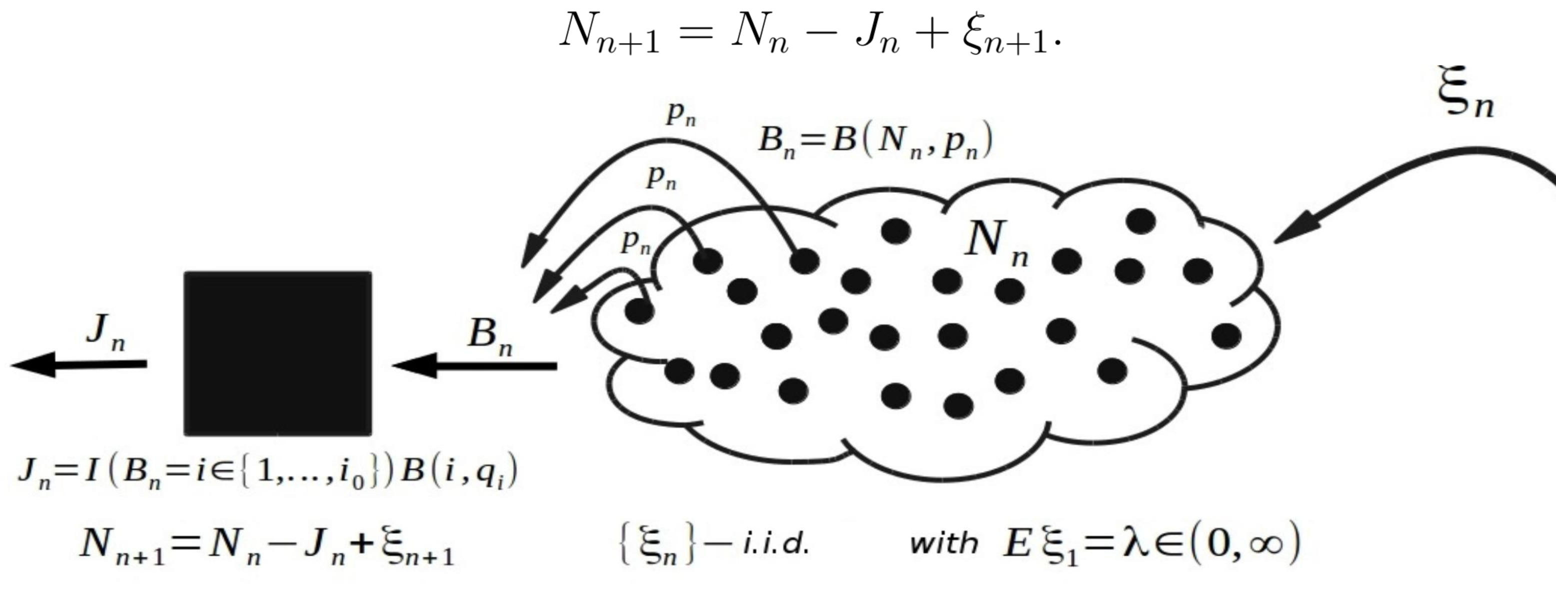
We consider a system with a non-standard partial feedback. Time is slotted. The numbers of arriving messages in different time slots are independent and identically distributed random variables. If $i \geq 1$ messages are being passed simultaneously, each of them is being passed successfully with probability q_i, and with probability $1 - q_i$ transmission is distorted, the message remains in the system and tries to be sent later. We consider the case when $q_i > 0$ only if $i \leq i_0$ for a given $i_0 \geq 1$. By the end of the slot we receive information about the quantity of messages that were transmitted successfully (it is the "feedback") – only this information is available.

The transmission algorithm (protocol) is a rule of setting transmission probabilities at different times based on the information, available to each moment. In particular, if $i_0 = 1$ and $q_1 = 1$ then we have "Success-Failure" binary feedback.

The input process of messages $\{\xi_n\}$ is assumed to be i.i.d., having a general distribution with finite mean $\lambda = \mathbf{E}\xi_1$, here ξ_n is a total number of messages arriving within time slot $[n, n+1)$.

So given that the total (unobserved) number of messages is N_n, the number of those B_n is sent to the channel, has conditionally the Binomial distribution $B(N_n, p_n)$ (here $B_n \equiv 0$ if $N_n = 0$). If $B_n = i \in \{1, ..., i_0\}$, then the total (observed) number of messages transmitted by the channel, J_n, also has conditionally the Binomial distribution $B(i, q_i)$. Otherwise $J_n = 0$, i. e. either the slot is empty ($B_n = 0$), or there is a message conflict ($B_n > i_0$). The following recursion holds:

$$N_{n+1} = N_n - J_n + \xi_{n+1}.$$



A transmission protocol is determined by sequence $\{p_n\}$. We consider "decentralised" protocols: the numbers N_n, $n = 1, 2, \ldots$ are not observable, and only values of past J_k, $k < n$ are known.

Let $N_1 \geq 0$ be the initial number of messages in the system and $S_1 \geq 1$ a positive number (which is an "estimator" of unknown N_1).

Let further $\mathcal{H}$ be a class of functions $h : [1, \infty) \to \mathbf{N}$ such that $h(t) \uparrow \infty$ is non-decreasing in t, and $h(t) = o(\sqrt{t})$ as $t \to \infty$.

With each $h \in \mathcal{H}$, we associate a class $\mathcal{E}_h$ of positive functions $\varepsilon_h : [1, \infty) \to (0, 1/2]$, such that $\varepsilon_h(t) \to 0$ and $h(t)\varepsilon^2(t) \to \infty$, as $t \to \infty$.

Let $C \in \mathbf{N}$ be a positive parameter, and let $\{I_n\}$ be an i.i.d. sequence that does not depend on the previous r.v.'s, with $\mathbf{P}(I_n = 1) = 1 - \mathbf{P}(I_n = 0) = 1/2$.

Let

$$p(z) \stackrel{def}{=} \sum_{i=1}^{i_0} \frac{q_i \cdot z^i}{(i-1)!} \quad \text{and} \quad \lambda_{max} \stackrel{def}{=} \max_{0<z<\infty} p(z)e^{-z}.$$

Consider the function $p(z)e^{-z}$ for $z \geq 0$. Let z_0 be the extreme right-hand point of the global maximum of the function $p(z)e^{-z}$; If $p(z^*)e^{-z^*} = \lambda_{max}$, then $z_0 \geq z^*$.

Also, for $t \geq 1$, we define

$$\beta(t) \stackrel{def}{=} 1 - \varepsilon_h(t) \quad \text{and} \quad \alpha(t) \stackrel{def}{=} 1 - \frac{1}{1 + \beta^{i_0}(t)\exp(z_0\varepsilon_h(t)/\beta(t))}.$$

The class $\mathcal{A}$ of algorithms is determined by C, h, ε_h, $\{J_n\}$ and $\{I_n\}$ as follows. The transmission probabilities p_n and the numbers S_n are updated recursively: given S_n and $J_n \in \{0, 1, ..., i_0\}$, we let

$$p_n = \begin{cases} \beta(S_n)/S_n, & \text{if } I_n = 0, \\ 1/S_n, & \text{if } I_n = 1, \end{cases}$$

and then with the parameter $\alpha(S_n)$ we play the Bernoulli random variable θ_n, and we let

$$S_{n+1} = \begin{cases} S_n + C, & \text{if } J_n \neq i_0, \\ S_n + h(S_n), & \text{if } J_n = i_0 \text{ and } I_n + \theta_n = 0, \\ S_n, & \text{if } J_n = i_0 \text{ and } I_n + \theta_n = 1, \\ \max(S_n - h(S_n), 1), & \text{if } J_n = i_0 \text{ and } I_n + \theta_n = 2. \end{cases}$$

We denote such an algorithm by $A(C, h, \varepsilon_h) \in \mathcal{A}$.

Definition. Algorithm A is stable if the underlying Markov chain (N_n, S_n) determined by A is ergodic, and unstable if the underlying Markov chain is transient.

Here are main results.

Theorem 1. *There exist $C \in \mathbf{N}$ such that, with any function $h \in \mathcal{H}$ and any function $\varepsilon_h \in \mathcal{E}_h$, algorithm $A(C, h, \varepsilon_h)$ is stable for any input rate $\lambda < \lambda_{max}$.*

Theorem 2. *Let $\lambda < \lambda_{max}$. If for some $\gamma > 1$ the power moment $\mathbf{E}\xi_1^\gamma < \infty$ is finite, then there exists a unique invariant distribution X such that*

$$\sup_B |\mathbf{P}(X_n \in B) - \mathbf{P}(X \in B)| \leq C_1/n^{\gamma-1},$$

where C_1 is some constant, and the supremum is taken over all sets $B \in \mathbf{Z}_+^2$ in the following two cases:

(a) for stable algorithms from the class $\mathcal{A}$, if $\gamma \geq 2$,

(b) for stable algorithms from the class $\mathcal{A}$ in which $h(t)$ is a slowly varying function, if $\gamma < 2$.

The last point of Theorem 2 remains valid for $h(t) = o(t^{1-1/\gamma})$, as $t \to \infty$. In particular, if the value γ is not known to us, then we can take $h(t)$ as a slowly varying function.

Theorem 3. *If $\lambda > \lambda_{max}$ and $\mathbf{P}(\xi_1 \geq i_0 + 1) > 0$, then any Markov "ALOHA protocol" is unstable.*

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