On Strong Law of Large Numbers for L-Statistics with Dependent Data

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The goal of the present note is to show that SLLN holds for L-statistics when data are stationary and ergodic sequences and also when data are stationary and ϕ-mixing sequences.

Let X_1, X_2, ... be a strictly stationary and ergodic sequence of random variables with a common distribution function F. We consider the L-statistics of the form

$$L_n = \sum_{i=1}^{n} c_{ni} h(X_{n:i}),$$

where $X_{n:1} \le \ldots \le X_{n:n}$ are the order statistics based on a sample $\{X_i; i \le n\}$, h is a measurable function, c_{ni}, $i = 1, \ldots, n$, are known constants.

Let us consider a sequence of functions $c_n(t) = nc_{ni}$, $t \in ((i-1)/n, i/n]$, $i = 1, \ldots, n$, $c_n(0) = nc_{n1}$. Define the parameter $\mu_n = \int_0^1 c_n(t)H(t)\,\mathrm{d}t$, where $H(t) = h(F^{-1}(t))$ and F^{-1} is the quantile function corresponding to the distribution function F.

Theorem 1. *If any of the following three conditions hold:*
(i) H *is continuous and* $\sup_{n\ge1}\sum_{i=1}^{n}|c_{ni}| < \infty$*;*
(ii) $\mathbf{E}|h(X_1)| < \infty$ *and* $\max_{i\le n}|c_{ni}| = O(1/n)$*;*
(iii) $\mathbf{E}|h(X_1)|^p < \infty$ *and* $\sup_{n\ge1} n^{q-1}\sum_{i=1}^{n}|c_{ni}|^q < \infty$ *(*$1 < p < \infty$, $1/p + 1/q = 1$*),*
then, as $n \to \infty$,

$$L_n - \mu_n \to 0 \quad a.s.$$

Recall that the stationary sequence $\{X_i\}$ is called ϕ-mixing if $\phi_n \to 0$, where

$$\phi_n = \sup_{k\ge1}\sup\{\,|P(B|A) - P(B)|; A \in \mathcal{F}_1^k, B \in \mathcal{F}_{k+n}^{\infty}\}$$

and $\mathcal{F}_a^b$ denotes the σ-field generated by X_a, X_{a+1}, ..., X_b.

Theorem 2. *Let* $\{X_i\}$ *be a strictly stationary and* ϕ*-mixing sequence with* $\sum_{i\ge1}\phi_i < \infty$*. Under the assumptions of Theorem 1, we have that the statement*

$$L_n - \mu_n \to_{a.s.} 0, \quad n \to \infty,$$

holds.