Limit theorems for the Lorenz-type curves based on dependent data

Evgeny Baklanov

Department of Mathematics, Novosibirsk State University, Pirogova 2, 630090, Novosibirsk, Russia
[baklanov@mmf.nsu.ru]

2000 Mathematics Subject Classification: 60F15, 62G30

Let $X_1,\ X_2,\ \dots$ be a φ-mixing sequence of random variables with a common distribution function F. We consider the Lorenz-type curve of the form

$$L(t) = \int_0^t J(y)h(F^{-1}(y))dy, \quad t \in [0,1],$$

where J is a Lebesgue-integrable function, h is a measurable function and F^{-1} is the quantile function corresponding to the distribution function F. Note that if $J \equiv 1$, $h(x) = x$, then $L(t)$ is a generalized Lorenz curve. The goal of the present report is to investigate the convergence, when $n \to \infty$, of the empirical Lorenz-type curve

$$L_n(t) = \int_0^t J(y)h(F_n^{-1}(y))dy, \quad t \in [0,1],$$

to the theoretical one $L(t)$, where F_n^{-1} is the quantile function corresponding to the empirical distribution function F_n. Define the function $H(t) = h(F^{-1}(t))$.
Theorem. *Let $\{X_i\}$ be a φ-mixing sequence with a common distribution function F with $\sum_{n\geq 1} \varphi^{1/2}(2^n) < \infty$. If any of the following three conditions hold:*
(i) *H is continuous and $\int_0^1 |J(t)|dt < \infty$;*
(ii) *$\mathbf{E}|h(X_1)| < \infty$ and $\sup_t |J(t)| < \infty$;*
(iii) *$\mathbf{E}|h(X_1)|^p < \infty$ and $\int_0^1 |J(t)|^q dt < \infty$ ($1 < p < \infty$, $1/p + 1/q = 1$),*
then, as $n \to \infty$,

$$\sup_{0\leq t\leq 1} |L_n(t) - L(t)| \to 0 \quad a.s.$$

In particular, under the assumptions of Theorem, we obtain SLLN for L-statistics:

$$\frac{1}{n}\sum_{i=1}^{n} c_{ni}h(X_{n:i}) \to \int_0^1 J(t)H(t)dt \quad a.s.,$$

where $c_{ni} = n\int_{(i-1)/n}^{i/n} J(t)dt$. Note also that Theorem remains valid for the empirical Lorenz-type curve based on stationary and ergodic sequences.